

A problem in elementary geometry

The authour believes that the following proposition belongs to elementary geometry, but he has no idea as to prove it without resorting to point-set topology.

Proposition 1. *Let X be a compact surface in \mathbb{R}^3 . If the section of X by any plane is a circle or a point, then X is a sphere \mathbb{S}^2 .*

Proof. Since X is compact, the set of circles of sections by planes has a maximum. Pick a pair of antipodal points N and S on the maximal circle, and consider the sections by planes passing through N and S . Then all such sections must be circles of the same radius. Thus X must be a sphere \mathbb{S}^2 . \square