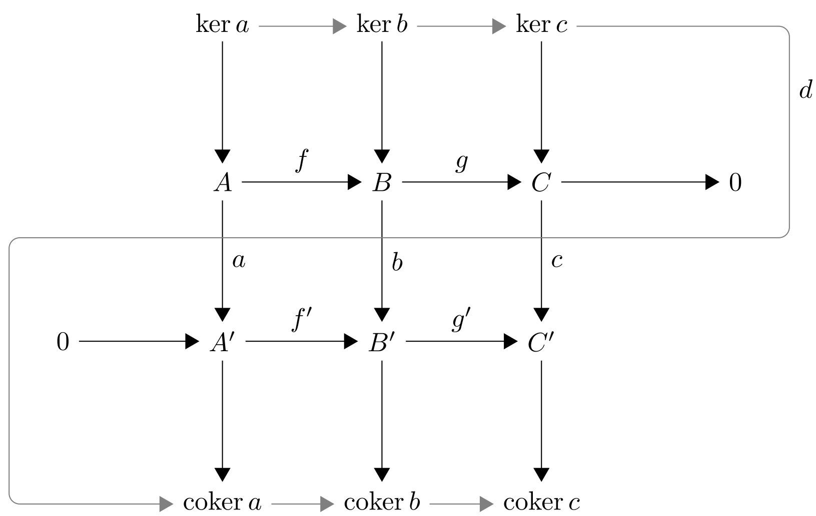
A “visual” proof of Snake Lemma

In Charles A. Weibel’s book, “An Introduction to Homological Algebra” (Cambridge U. Press, 1994), there appears the following: “We will not print the proof (of the Snake Lemma) in these notes, because it is best done visually. In fact, a clear proof is given by Jill Clayburgh at the beginning of the movie *It’s My Turn*.”

Following the suggestion by Weibel, we transcribe Jill Clayburgh’s proof of the “Snake Lemma” as in https://youtu.be/etbcKWEKnvg.



KateGunzinger: Let me just show you how to *construct* the map d, which is the fun of the lemma anyhow, okay? So you assume you have an element in the kernel of c, that is, an element in C, such that c takes you to 0 in C'. You pull it back to B, via map g, which is surjective...

Cooperman: Hold it, hold it, hold it. That’s – that’s not unique.

KateGunzinger: Yes, it is unique, Mr. Cooperman. Up to an element of the image of f, all right? So we’ve pulled it back to a fixed B here. Then you take β of B, which takes you to 0 in C', by the commutativity of the diagramme. It’s therefore in the kernel of the map g', hence is in the image of the map f', by the exactness of the lower sequence...

Cooperman: No.

KateGunzinger: ...so we can pull it back...

Cooperman: No.

KateGunzinger: ...to an element in A'...

Cooperman: It's not well defined!

KateGunzinger: ...which it turns out is *well* defined *modulo* the image of a. And thus defines the element in the co-kernel of a ...
—draws arrow on diagram—
and that's the "snake"! And on Monday, we'll address ourselves to
—Cooperman raises hand—

KateGunzinger: the cohomology of groups... and Mr. Cooperman's next objections.