

Russell's paradox

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Russell's paradox gives a contradiction arising from regarding every collection as a set. This article is a slightly modified version of the exposition by J. J. Rotman (in 'Introduction to Homological Algebra'.)

Definition 0.1. A set S is called a *Russell set* if it is not a member of itself; that is, $S \notin S$.

Let R be the collection of all Russell sets, then either

- R is a Russell set, or
- R is not a Russell set.

Suppose R is a Russell set, then $R \notin R$, by definition. But all Russell sets lie in the collection of all Russell sets, namely, R , so $R \in R$. Thus we have a contradiction.

Now, suppose R is not a Russell set, then R does not lie in the collection of all Russell sets; that is, $R \notin R$. But, then R satisfies the criterion for being a Russell set. Thus we have another contradiction.